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The Impact of Expected Return Uncertainty on Long-Term Risk and Investment Allocation Decisions

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Uncertainty in expected return estimates introduces a risk that is small over short periods of time and therefore often ignored. However, this risk becomes very important over longer horizons, in some circumstances outweighing more conventional risks. This paper demonstrates the effect of incorporating uncertainty of expected returns into an asset allocation process. It builds on earlier studies that demonstrated the increased effect of uncertainty as the investment horizon lengthens. This analysis is extended to active management or hedge fund allocations, where uncertainty is arguably greatest.

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“Doubt is uncomfortable, but certainty is ridiculous.”

Voltaire

“To know that we know what we know, and to know that we don’t know what we don’t know, that is true knowledge.”

Copernicus

Introduction

The conventional investment-allocation process makes recommendations based upon the expected return and variability of the possible investments. Though these inputs are estimates, there is usually no explicit attempt to factor possible errors in those estimates into the analysis. This paper provides a framework for incorporating uncertainty in expected returns into an optimization. For long-horizon decisions, uncertainty in expected returns can be quite important. The paper contrasts the effect of uncertainty with that of mean reversion in stock returns and notes that the effects work in opposite directions.

Uncertainty is much larger for active-management strategies than for asset classes because the investments are less transparent, more dynamic and have shorter histories. Recent innovations allow investors to take more active risk. In such cases, it is particularly important to recognize the potential error in expectations. If you do not, active strategies will appear less risky, and therefore more attractive, than they truly are. The last sections of this paper sketch out a framework that, if fully developed, can help identify better allocations for long-term investments.

To set the stage, consider a choice posed by Daniel Ellsberg (1961).[1],[2] Suppose that there are two urns. Urn 1 contains 50 red balls and 50 black. Urn 2 contains 100 balls that are either red or black, but you are not told the proportion[3]. To assure that the game is honest, you can pick the color to bet on. The payoff will be $70 if you randomly draw your color ball from one of the urns and -$30 if you draw the other color. Since the probabilities are symmetric but the payoffs are not, the game has positive expectation. The question Ellsberg asked was “Would you rather draw from Urn 1 or Urn 2, or are you indifferent?” If you are like most people, you have a definite preference to draw from Urn 1. This appeared paradoxical because the odds are 50/50 from either urn. He posited aversion to ambiguity that was distinct from risk aversion.

However, if you consider multiple trials, the common choice is not paradoxical at all. Arguably, your best estimate of the distribution of outcomes is identical for one draw. For both urns, you believe there is a 50% chance of red, 50% of black. However, if we

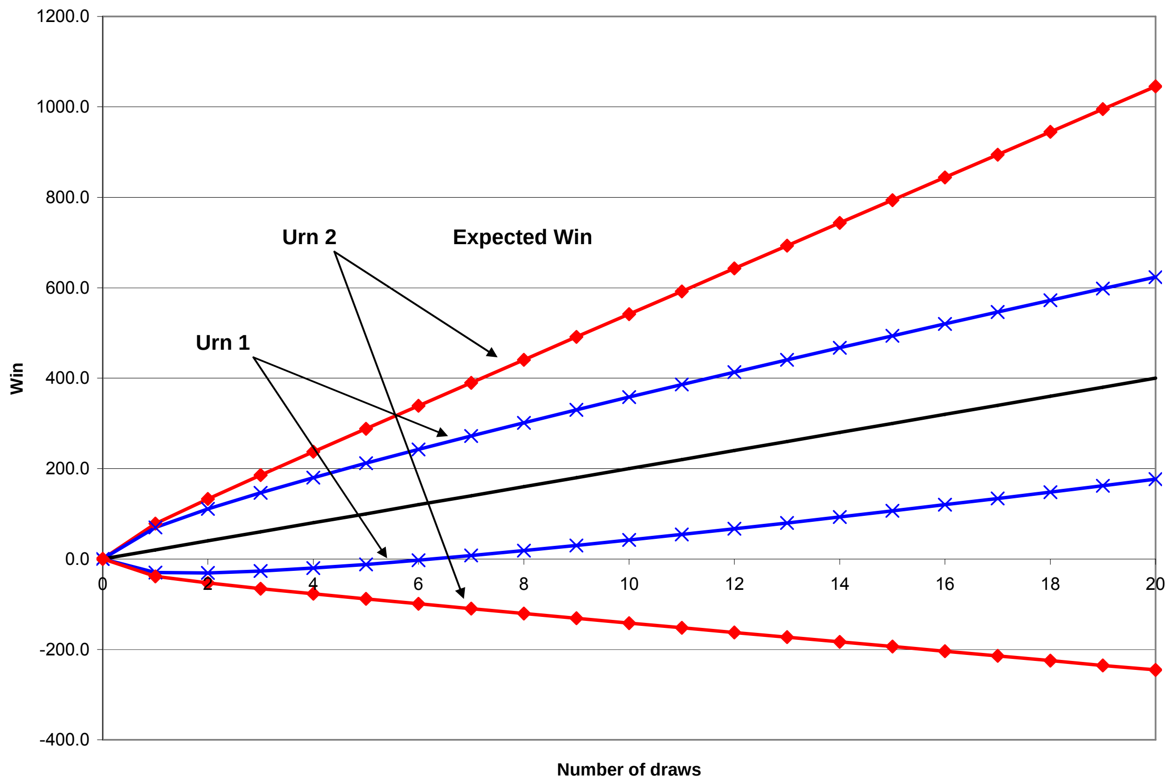
[1] For readers old enough to remember him: yes, the Daniel Ellsberg of the Pentagon Papers..

[2] This is not exactly the question Ellsberg posed but is very similar. Frank Knight (1921) posed a nearly identical question. However, he supposed that the “man would have to act on the supposition that the odds were equal” and did not consider the matter further (page 219). Chipman (1961) independently performed similar (but more careful) experiments and found the same behavior as Ellsberg.

[3] Assume that someone randomly drew an integer “N” between 0 and 100 and put N red balls and (100-N) black balls in Urn 2.

consider repeated draws (replacing the balls in the urns), the odds of a streak of red or black is higher for Urn 2 that for Urn 1. Our estimate of the long-term distribution for Urn 2 is much more diffuse that for Urn 1. Figure 1 displays the +/- one standard deviation range of your profit for each urn as the number of trials increases. If you play the game with Urn 1, your odds of winning steadily increase towards certainty. That is not the case with Urn 2. It is possible that the odds are actually unfavorable and you will lose in the long run.[4] So, risk aversion is enough to explain a preference for Urn 1 as long as you contemplate multiple trials. More importantly, it shows how the time horizon (number of trials) affects your decision.

Figure 1
+/- 1 Standard Deviation Range of Outcomes



Draws from Urn 1 are uncertain in the sense that we don't know whether any one draw will be red or black. However, we do know the odds. This is the type of risk we traditionally recognize. I will it "variability risk". For Urn 2, we do not even know the odds. I will call this additional uncertainty "expectations risk".[5] The total risk includes both expectations and variability risk. This example illustrates how the impact of expectations risk becomes more important over longer horizons. This paper will examine this phenomenon and its impact on investment decisions.[6]

[4] Viewing it as a one-period problem, having a preference for Urn 1 (and similar choices discussed in Ellsberg (1961)) seems to violate a basic principle of rational decision-making (Savage's Sure Thing Principle) and this was dubbed the "Ellsberg Paradox". However, in a more general multi-period context, preferring Urn 1 is quite rational. See also Segal (1985).
[5] The reasons for this terminology will be discussed in the next section.
[6] Errors in variances and covariances matter too. But preliminary investigation suggests that the impact is smaller (Chopra and Ziemba (1993)). Nonetheless, further study of this is warranted.

One common response to this game is that each draw from Urn 2 provides information. Over time, or so goes the theory, you would develop a good estimate of Urn 2's contents and make very effective bets. This is true of the artificial example. In more realistic circumstances portfolio managers are likely to encounter, I assert that learning does not occur quickly enough to change the results substantially. Sections 7 and 9 of this paper support this assertion.

1. Previous Work on Investments and Uncertainty

Bawa, Brown and Klein (1979), Jobson and Korkie (1981), Gennotte (1986), Frost and Savarino (1986), Black and Litterman (1990), Michaud (1998), and Kandel and Stambaugh (1996), among others, explored estimation risk and its impact on investments. However, they addressed only single-period optimizations.[7],[8]

More recently, Barberis (2000), Scherer (2002) and Brennan and Xia (2001) studied the impact of uncertainty over longer periods of time on optimal allocations. Their results are quite similar to mine. This paper examines how expectations risk impacts total risk and how that impact increases over time. Unlike previous work it examines expectations risk for both asset classes and active strategies and places them in a combined framework. This paper attempts to be simpler and more practical than previous work.

In addition, past work has generally considered expectations risk to be the error in statistical estimates – that is, estimation risk. This paper recognizes that we often arrive at estimates based on more information than just the historical record. Uncertainty, therefore, is not simply a statistical phenomenon. In addition, I'd emphasize that I am considering only errors in expected returns and not those relating to variance/covariance estimates or model risk. For these reasons, I use the term "expectations risk". For brevity and variety, I will sometimes also use Ellsberg's term, "ambiguity".

In addition, this paper will address expectations risk regarding active strategies to add value. There is some useful work in this area (Baks, Metrick and Wachter (2001) and Cvitanic, et al (2003)) but it has not considered long horizons. Empirical work in this area is made both more difficult and more important by the lack of good historical data.

2. Simplest Case

Let's explore the simplest case. Consider two potential investments: cash and stocks. Cash has an expected return of 4% and zero risk.[9] Stocks have an expected return of 8% and a risk of 16%. Since Markowitz (1952), the conventional approach to problems of this sort is to map out an efficient frontier. Two questions are rarely asked: "How reliable are the inputs?" and "What is the time horizon?".[10] It turns out that these questions matter a great deal and are interrelated.

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[7] Michaud (1998) is an excellent exposition of expectations risk in general. Unfortunately, while it briefly considers multiperiod optimizations in Chapter 3, it does not address the issues addressed in this paper.
[8] Black and Litterman explicitly allow for subjective estimates of uncertainty. But they only use it to adjust expectations. It is not incorporated into risk.
[9] This is a gross oversimplification for expositional reason. In fact, fixed income risk is not zero and actually has more complex time dynamics that other asset classes. This is a field in need of significant study. An excellent start has been made in Cambpell and Viceira (2005)
[10] How many efficient frontiers have you seen that specify the time horizon they apply to?

Samuelson (1969) and Merton (1969) showed that the optimal allocation is the same for all time horizons *under certain conditions.* One of the most crucial conditions is that successive returns are independent of each other. Merton (1973) (and Samuelson (1989)) noted that if returns are not independent over time, the optimal allocation varies with the investor's horizon. However, conventional practice has embraced the original results but has generally ignored the qualification. The next two sections explore how risk grows over time, how expectations risk affects that growth, and how this, in turn, impacts optimal allocations.

3. Increase in Dispersion over Time

To get insight into how risk grows over time, it is useful to consider a discrete two-year case. Suppose that stock returns are generated by the following process:

$$R_t = \mu + \varepsilon r_t, \qquad \text{(Base Model)} \qquad (1)$$

where t designates time, μ is the expected excess return (that is, over the riskless asset). εr_t is a random variable with a mean of zero and standard deviation of σr.

The two-year cumulative return (ignoring compounding) is

$$= \mu + \varepsilon r_1 + \mu + \varepsilon r_2 \qquad (2a)$$

and the expected cumulative return is:

$$= 2\mu \qquad (2b)$$

because the expected values of εr_1 and εr_2 are zero.

The two-year variance is

$$\sigma r^2 + \sigma r^2 + 2\,\rho_{1,2}\,\sigma r^2, \qquad (3)$$

where $\rho_{1,2}$ is the correlation between returns in years one and two.

You can see that if $\rho_{1,2} = 0$ (R_1 and R_2 are independent of each other), then the two-year variance is $2\,\sigma r^2$. The reason the optimal allocation is independent of horizon is that both the expected return and the variance grow linearly with time.[11] This is consistent with Merton and Samuelson's results.

However, if $\rho_{1,2}$ is negative, returns are mean-reverting. If this is so, the two-year variance is less than twice the one-year variance: Risk grows more slowly than time. Conversely, if $\rho_{1,2}$ is greater than zero, then risk grows more quickly than time.

4. Expectations Uncertainty and Its Impact over Time

But we do not know μ. Supposing that the base model is the true return generating process, we still need to recognize that we only have an estimate of μ. So, from our perspective, the stock-return generating process is:

$$R_t = \mu_e + \varepsilon m + \varepsilon r_t, \qquad \text{(Base Model With Ambiguity)} \qquad (4)$$

[11] This is not strictly true for compounded returns but it is true that the optimal allocation does not vary with horizon.

where μ_e is our estimate of the mean return ($\mu = \mu_e + \varepsilon m_t$), and εm is a random variable with a mean of zero and standard deviation of σm, representing the error in our estimate.

The two-year variance is

$$2\ \sigma r^2 + 2\ \sigma m^2 + 2\ \rho m\ \sigma m^2 \quad (5)$$

ρm represents the correlation in errors in the mean from period to period. Without learning, $\rho m = 1$ and so the two-year variance is:

$$= 2\ \sigma r^2 + 4\ \sigma m^2 \quad (6a)$$

More generally, the T-year variance is

$$T\ \sigma r^2 + T^2\ \sigma m^2 \quad (6b)$$

Expectations risk grows with the horizon squared. Typically, σm is smaller than σr. As a result, it is relatively unimportant for short horizons. However, because it grows with time squared, it become increasingly meaningful as the horizon lengthens. For illustration, let's use the parameters mentioned earlier: excess return $\mu_e = 4\%$, variability risk $\sigma r = 16\%$ and expectations risk $\sigma m = 2\%$. Table 2 displays the risk of cumulative returns for one- and ten-year horizons. You can see that expectations risk contributes very little to the overall risk for one year. By ten years, it is material. When we turn to active management, we will see it is even more important.

Table 1
Impact of Expections Risk on Total Risk

	Variability Variance	Expectations Variance	Total S.D. (annualized)	Expectations/Total
1-Year	0.0256	0.0004	16.1%	1.5%
10-Years	0.2560	0.0400	17.2%	13.5%

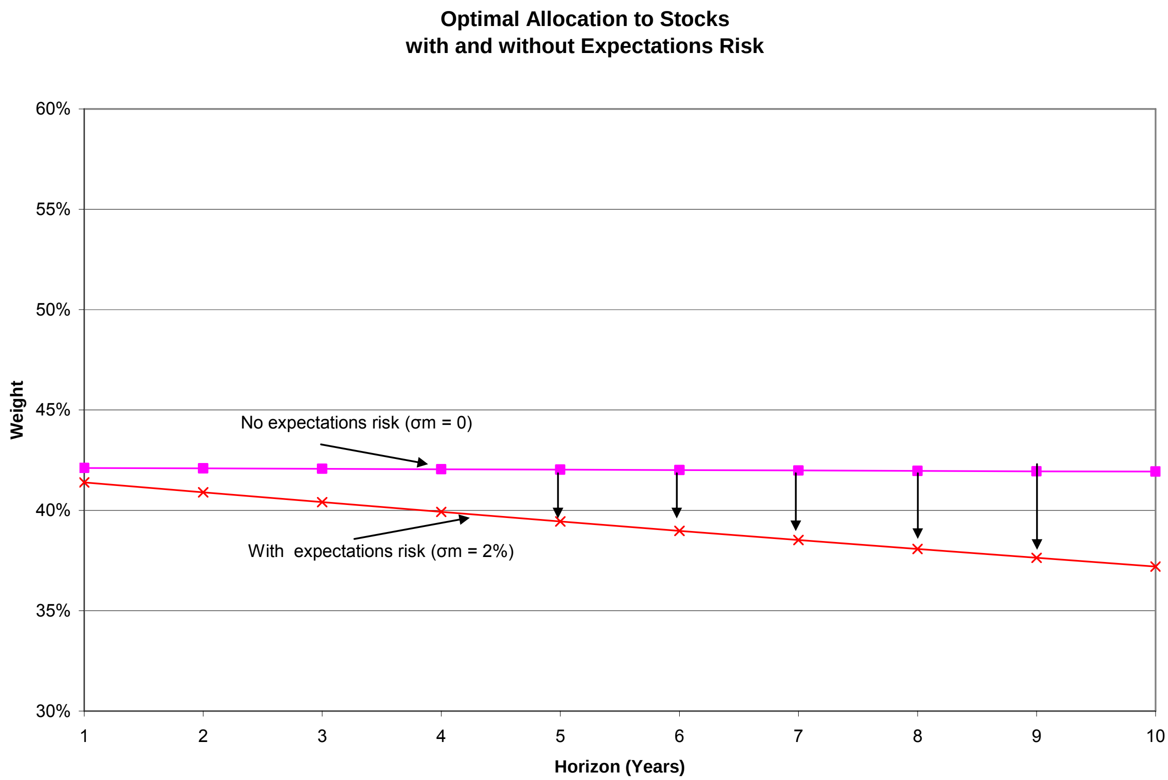
Since expectations risk is persistent, time horizon matters. I will use a particular optimization process and level of risk aversion (details in footnote 12). However, the results are qualitatively the same for many processes and risk levels. Figure 2 shows the optimal allocation to stocks.[13] The arrows indicate the impact of ambiguity. You can see that the optimal portfolio is constant when ambiguity is not considered. However, when expectations risk is recognized, the optimal allocation to equities declines materially as

[12] These optimizations maximize expected utility using the Constant Relative Risk Aversion (CRRA) utility function $U(W) = W^{(1-\gamma)} - 1)/ (1 - \gamma)$ with $\gamma = 4$. This is equivalent to mean-variance optimization (in particular maximizing Expected Return – $(\gamma/2)$ X Variance) if returns are normally distributed and close under more general assumptions. I assume that both εr and εm are binomial. This enables me to enumerate all of the possible paths and calculate the optimum directly. Barberis (2000) simulates a continuous distribution 1,000,000 times and arrives at very similar results. See Campbell and Viceira (2002) and Uysal, Trainer and Reiss (2000) for discussions of these are related issues. Campbell and Viceira (chapter 2) also contains a good discussion of how (and under what circumstances) time horizon matters.

[13] I continue to ignore a number of "real-world" complications regarding fixed income. The riskless asset changes when the horizon changes (and depending on whether real or nominal risk is being considered. The impact of horizon on fixed income risk is more important and more complicated than for equities. Arguably, all of the risk of T-bills is expectations risk. This is an important, but separate, phenomenon. I will ignore it and recommend that the reader refer to Trainer, et al (1979) and Campbell and Viceira (2002, Chapter 3 and 2005).

the horizon lengthens. This is not surprising since ambiguity grows as the square of time while the expected return is still a linear function.

Figure 2



The data shown in Figure 2 are not new. This result replicates those of Barberis (2000), Scherer and Martin (2005) and others. Barberis extends this to a more realistic model where stock returns are mean-reverting. The next section examines this process.

5. Predictability and Its Impact Over Time

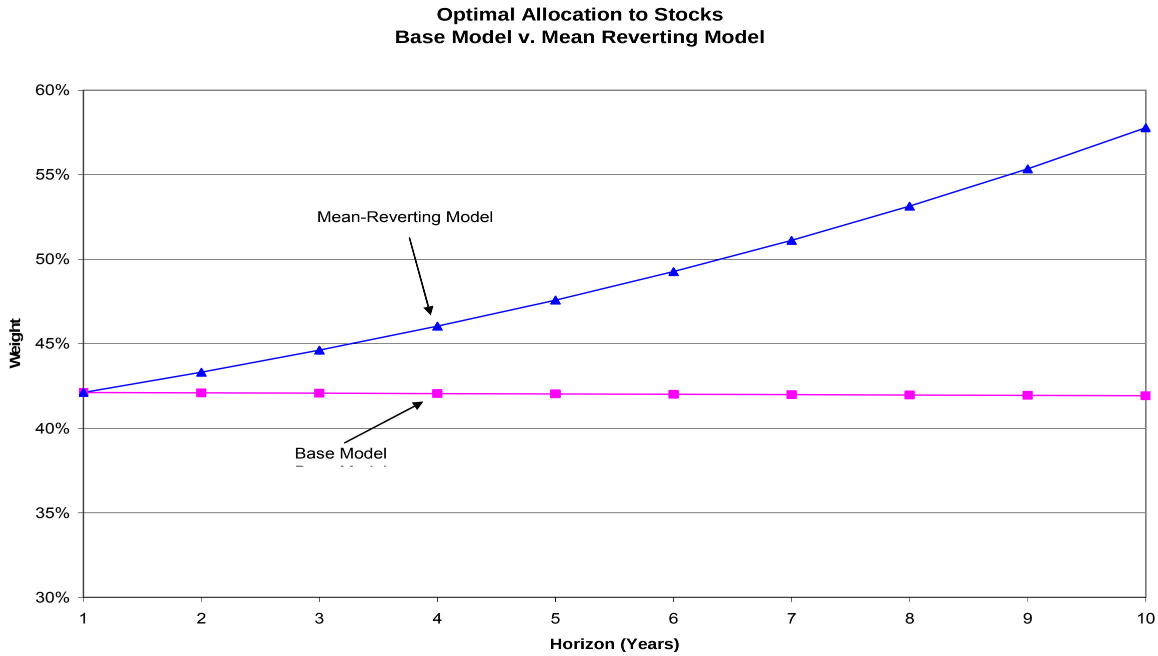
An important counterpoint to the effect of expectations risk is predictability. If stock returns are mean-reverting, this will cause risk to grow more slowly. In fact, there is significant (but uncertain) evidence that they are. Campbell and Shiller (1988 and 2001) among others provide empirical evidence that stock-market returns are mean-reverting. They give evidence that high dividend yields forecast high returns. Since dividends are much more stable than prices, falling prices raise dividend yields, which in turn forecast better returns: a mean-reverting process. To make this tangible, suppose that the stock-return process is as follows:

$$R_t = \mu + b\,(d_t - D) + \varepsilon r_t, \qquad \text{(Unambiguous Mean-Reverting Model)} \quad (7)$$

where d_t is the dividend yield at time t, D is the "normal" dividend yield and b is the impact of dividend yield on return.

For now, I'm assuming that we know the true process. Expectations risk will be incorporated shortly. Under this process[14], ρr is negative and so risk grows less than linearly with time. In this case, the optimal allocation to stocks will increase with the horizon. Figure 3 contrasts the Unambiguous Mean-Reverting Model's allocation assuming μ = 8%, B=3, D=2%, and σr = 16% with the Base Model's.

Figure 3



6. Bringing ambiguity back into model

Of course, we cannot be certain of the parameters of the Mean-Reverting Model either. Goyal and Welch (2002) for example, assert that dividend yields have no predictive power (that is, that the Campbell and Shiller result is a statistical fluke. So, it seems sensible, if we are going to model mean reversion, to recognize that we don't know the precise parameters of the process. The model with expectations risk is:

$$R_t = \mu_e + \varepsilon m_t + (b_e + \varepsilon b) \times (d_t - D) + \varepsilon r_t, \qquad \text{(Ambiguous Mean-Reverting Model) (8)}$$

where b_e is the estimated value of b, and εb is the error in that estimate with standard deviation σb.[15]

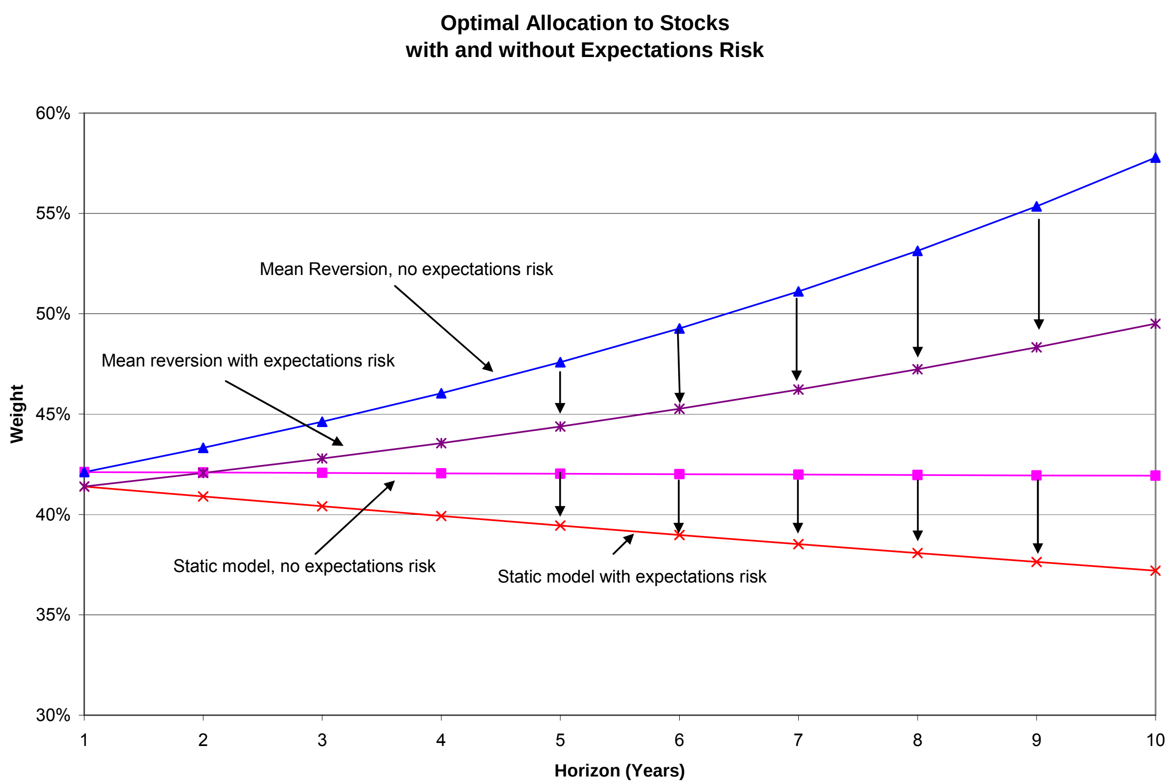
The Ambiguous Mean-Reverting Model includes error regarding both the mean and the predictive value of the dividend yield. The parameters were chosen to be roughly consistent with Barberis's empirically estimated values. Figure 4 contrasts the optimal allocation for the Mean-Reverting Model for a stock with initial dividend yield D, with and without incorporating ambiguity effects. As in Figure 2, the arrows show the impact of

[14] And assuming that the actual dollar dividend is known and grows annually by $\mu - D$%. Dividends grow at the same rate as the expected growth of stock prices, keeping the dividend yield constant.
[15] A separate error term for D is redundant; εm represents error in both μ and D.

adding ambiguity. As you can see, ambiguity decreases the optimal allocation by an increasing amount over time. But both here and in Barberis's much more rigorous study expectations risk only partially offsets the effect of mean reversion for horizons out to 20 years.[16] That is, the optimal allocation to stocks increases with the horizon in the Uncertain Mean-Reverting Model, despite our uncertainty about both the mean and the importance of dividend yields.

Thus far I have ignored two effects of the passage of time. First, over time, we accumulate more data and, perhaps, are able to improve our estimates—that is, we learn. Second, the dividend yield would change, causing us to adjust our allocation for the mean-reverting models. Space doesn't permit extensive discussion of these issues here. Barberis (2000) found that these effects do not change the qualitative conclusions. I will discuss learning briefly in the next section.

Figure 4



7. Estimating Expectations Risk

Expectations risk is inherently difficult to estimate. To the extent that your estimates are quantitatively derived, there are natural statistical measures of parameter uncertainty (see Scherer and Martin (2005) and Barberis, among others). But considering that ambiguity is primarily evident over long periods, the available history is really too short to rely heavily on those estimates. To expand the sample, it is useful to go back far in

[16] Eventually, expectations risk would become more important than mean reversions

time and to look across multiple countries. Jorion and Goetzmann (2000) and Dimson, March and Stanton (2002) have done an invaluable service compiling a wealth of information.

But even this has its limitations. Clearly, the markets have changed in many ways and there are biases to the history (Brown, Goetzmann and Ross (1995) and Goetzmann (2000)). We should not forget the stock market results in the 1910s, 20s and 30s (and certainty not those of Japan in the 1990s) but it is not clear how relevant they are. In addition, there is So, judgment is important both in arriving at the expectations and in assessing the amount of expectations risk. AIMR and TIAA-CREF assembled a panel of experts and a fascinating review of the topic is contained in TIAA-CREF (2001). Ultimately, each of us needs to make an informed and conservative judgment. My assessment for the stock market is 2%. It is certainly significantly greater than 0.

Considering the level of expectations risk, even when we are in possession of 100 years of global data, it is clear that ambiguity will not go away. I have assumed that the level of ambiguity is constant and I think that is reasonable. At best, it will decline slowly. Barberis (2000) has modeled learning with uncertainty and his results are qualitatively similar to those without learning.

8. Ambiguity and Active Management

Expectations risk is even more important when considering active management. This is particularly true because many investors are increasing the share of the risk budget they are allocating to active management.

Traditionally, active management decisions have been tied to asset allocation decisions. An investor (particularly pension plans, etc.) decides how to allocate among broad asset classes and then looks for good managers within those asset classes (a good example is Figelman (2004). This constrains the amount (and types) of active management.

The idea of portable alpha is that it is possible to use derivatives, leverage and shorting to separate the investment management skill (alpha) from the asset class. This allows the investor to "port" the alpha wherever desired. It also removes the constraints on active management.

Portable alpha is a useful way to look at the problem conceptually whether the investor wants to implement it this way or not. It makes the investor aware of any tension that might exist between the asset allocation decision and the active management on. This framework has the potential to substantially improve the return/risk profile for the investor. Litterman (2004) and Kritzman and Thomas (2004) argue that this freedom should be used to increase the risk allocated to active management- in some cases, enormously so.

However, this *presumes* that the investor can identify managers with positive alpha. The records of active management are shorter than those for asset classes. In addition, considering selection and other biases, it is an open question whether a manager's historical records have any predictive validity. Paul Samuelson (1989) (and many others) argue that it is very difficult to identify skillful managers. Further, he argues that the pursuit of alpha is a dangerous and irrational activity, akin to an alcoholic taking a drink.[17] If the investor takes this approach, he need not consider active management at all. However, many (most?) investors appear to be convinced not only that superior active managers exist but that they are able to identify them.[18] Even so, it seems only prudent to recognize that when we estimate an active manager's alpha, that estimate is uncertain.

Suppose that we have identified an active manager and we have estimated their expected alpha to be 3% and their variability risk to be 6%. If we assume no expectations risk, we are implicitly assuming that the manager is almost certain to outperform given enough time. But, if we acknowledge expectations risk, this is not the case. Table 2 shows the manager's odds of outperforming by horizon and amount of expectations risk.

Table 2
Probability That the Active Strategy Outperforms
by Expectation Risk and Horizon*

Horizon	0% expectations risk	4.5% expectations risk	9% expectations risk
1	69%	66%	61%
2	76%	69%	62%
5	87%	72%	63%
10	94%	73%	63%
20	99%	74%	63%

* assumed alpha = 3%, variability risk = 6%.

As we've seen before, the impact of expectations risk is small for short horizons but gradually becomes significant. The appropriate horizon will be discussed in section 9. The appropriate amount of expectations risk depends on the manager and the investor, but I would argue that it should never be zero. The middle column has expectations risk equal to 1.5 times the alpha. This means that underperforming over the long-term is a 2/3 standard deviation occurrence. This (assuming a normal distribution) is consistent with 75% confidence that you have selected a truly superior manager. The odds of outperformance with 4.5% expectations risk are 74% after 20 years and, over longer periods, reaches 75%. Considering the difficulty of active management and the difficulty of an investor identifying true skill, that seems like a reasonable level of ambiguity. Of course, each specific situation should be addressed individually but this is a reasonable[19] assumption to use to illustrate the framework.

Table 3 summarizes the assumptions I will use for optimizations that combine active and passive investments.

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[17] I would add another cautionary note that seems particularly relevant at the time of this writing. While skilled, honest managers can produce better returns in a portable-alpha or hedge-fund structure, less skilled and less honest managers can also prosper – at least long enough to get rich. In these conditions, it is far from clear that the quality of active management on average improves.

[18] The author confesses to being in this group. I don't let Samuelson's sound advice interfere with my plans.

[19] Actually, it is quite optimistic because it assumes you have identified a truly superior manager. This is about as optimistic as it is prudent to be. A more conservative assumption (a la Samuelson) would be a negative alpha (after fees) but then the question of allocation to the active manager would be moot.

Table 3
Assumptions for Active/Passive Optimization

	Expected Excess Return	Variability Risk (σr)	Expectations Risk (σm)
Stock	4%	16%	0 or 2%
Active	3	6	0 or 4.5

By construction, the correlations between the strategies and between expectations and variability risk are all zero. Note also that "Active" is a self-financing strategy. That is, I'm assuming that it does not consume any capital and that it can be leveraged to the extent desired. I assume mean reversion as above for stocks but no mean reversion for active strategy (I'll discuss that assumption in the next section).

Table 4
Optimal Portfolios for One-Year Horizon Without Expectations Risk[20]

	Case 1
Horizon	1-Year
Expectations risk used in optimization?	**No**
Optimal Allocations	
Stocks	42%
Active	225
Expected Return	**12.44%**
Total Variability Risk	**15.09%**
Stock Variability Risk	6.75
Active Variability Risk	13.50
Total Expectations Risk	10.16%
Stock Expect. Risk	0.84
Active Expect. Risk	10.12
Total Risk	18.19%

Table 4 shows the results of a conventional optimization – one that has a one-year horizon and ignores expectations risk. It allocates 42% to stocks and leverages the active strategy 225%. This results in an expected return of 12.44% and variability risk of 15.09%. Since the optimization is ignoring expectations risk, it is optimizing the numbers shown in bold. Of the active risk, 6.75% comes from stocks and 13.5% (twice as much) comes from the active strategy. This is the natural result of the input that the active strategy's information ratio is twice that of stocks. The portfolio appears to be attractive – *but only if you ignore expectations risk.* When we bring that factor into play, we see that the portfolio is much riskier than it first appears. The active strategy has

[20] For simplicity, mean-variance optimizations were used based on annualized risk. The risks listed are annualized standard deviation. The values actually used in the optimization are shown in bold.

nearly as much expectations risk as the conventional variability risk. So the total risk of the strategy is 18.19%.

Case 2 recognizes both expectations risk and variability risk. That is, it optimizes return versus total risk. Table 5 contrasts Case 2 with Case 1. The recommended portfolio still leverages the active strategy, but not as much. Naturally, when the optimizer is aware of expectations risk, it curtails the allocation to the active strategy.

Table 5
Optimal Portfolios for One-Year Horizon Without and With Expectations Risk

	Case 1	Case 2
Horizon	1-Year	1-Year
Expectations risk used in optimization?	**No**	**Yes**
Optimal Allocations		
Stocks	42%	41%
Active	225	160
Expected Return	**12.44%**	**10.44%**
Total Variability Risk	**15.09%**	11.66**%**
Stock Variability Risk	6.75	6.65
Active Variability Risk	13.50	9.57
Total Expectations Risk	10.16%	7.23%
Stock Expect. Risk	0.84	0.83
Active Expect. Risk	10.12	7.18
Total Risk	18.19%	**13.71**%

Table 6 adds the optimal portfolios for 10-year horizons.

Table 6
Optimal Portfolios for One and 10 Year Horizons

	Case 1	Case 2	Case 3	Case 4
Horizon	1 year	1 year	10 years	10 years
Expectations risk used in optimization?	**No**	**Yes**	**No**	**Yes**
Optimal Allocations	42%	41%	64%	53%
Stocks	42	41	64	53
Active	225	160	225	44
Expected Return	**12.44%**	**10.44%**	**13.29%**	**7.44%**
Total Variability Risk	**15.09%**	11.66%	**15.84%**	7.48%
Stock Variability Risk	6.75	6.65	8.29	7.00
Active Variability Risk	13.50	9.57	13.50	2.64
Total Expectations Risk	10.16%	7.23%	32.27%	7.11%
Stock Expect. Risk	0.84	0.83	4.02	3.35
Active Expect. Risk	10.12	7.18	32.02	6.27
Total Risk	18.19%	**13.71**%	35.95%	**10.32**%
Stock Total Risk	6.80	6.71	9.21	7.76
Active Total Risk	16.87	11.96	34.75	6.80

Note: Annualized risks and expected returns are shown.

Case 3 shows the optimal portfolio for a 10-year horizon but without expectations risk. The optimal equity allocation rises significantly because of mean reversion. Since I have not assumed any mean-reversion for the active strategy, its allocation is the same 225% as the one-year horizon (Case 1). But when we examine the expectations risk of this strategy, we can see that this portfolio is extremely risky in the long term. Basically, this portfolio is very exposed to the question of whether the active manager actually has skill. The long-term dispersion is equivalent to 35% annual variability if the returns were independent from year to year. I don't mean to suggest that the year-to-year variability will be anywhere near that level. However, because of the persistent nature of the expectations errors, the 10-year dispersion is as wide as a portfolio with 35% variability. We are in a situation much like Urn 2 shown in Figure 1.

When you allow the optimizer to consider long-term expectations risk (Case 4) it arrives at a less risky and more balanced allocation. Because of the ambiguity about whether the manager has skill, the allocation to that strategy drops form 225% to 44%. Of course, this also reduces the expected alpha significantly. Two rows have been added at the bottom of the table that show how risk is distributed between the investments. Interestingly, risk is distributed roughly equally between stocks (7.76%) and the active strategy (6.80%). For stocks, the big risk is variability, but expectations risk is material for this horizon. The active strategy is just the reverse. Its biggest risk is expectations. This is simply a reflection of the common sense fact that in the long run the key question in deciding on an active strategy is: "Does the manager actually have enough skill to overcome fees and costs?"

I assert that a long term optimization with expectations risk (Case 4) is what most investors should focus on. It's notable that Case 4 is actually most similar to the strategy followed by typical investors. Perhaps investors are positioned this way because, as some authors have suggested, convention and constraints have prevented more efficient allocations to active strategies. But it is also possible that an intuitive appreciation of expectations risk has been part of the reasons investors have not followed the logic of portable alpha to its "logical" conclusion. Because investors recognize this sort of risk, they do not use optimization frameworks at all or impose strict constraints on asset classes such as hedge funds. Even if investors have arrived at roughly the desirable solution intuitively, it doesn't follow that investors are just as well off without optimizations. The typical situation faced by investors involves a large number of asset classes and an even larger number of potential active strategies. It seems clear that a careful assessment of all of the risks that investors face – including expectations risk – and a systematic framework that incorporates those risks should result in more reasonable allocations.

9. Practical Issues Regarding Implementation

I argued in Section 7 that learning about asset classes is a slow enough process that it can be ignored. This is clearly not the case for active strategies. The expectations risk I assess for active strategies is equivalent to about three years of track record. This is not because active managers' histories are necessarily that short. It reflects the dynamic nature of the strategies, their lack of transparency, and the fact that the managers are looking for inefficiencies that "should" not be there. In addition, there is substantial selection bias. All of these factors make it more difficult to assess an active manager's skill and suggest using a high level of ambiguity.

In practice, the amount of expectations risk will vary from manager to manager. It clearly is related to the amount of active risk or tracking error the manager takes. But there are other considerations, too. The transparency of the process, degree of leverage, type of decision-making process, length of the manager's record all may provide someinsight. Looking at cross-sectional risk across similar managers may also help. Baks, Metrick and Wachter (2001) have an interesting approach to this question.

Considering the high expectations risk, each year's return provides a significant amount of new information. The question is: "What's the best use of the information?" Fundamentally, many active strategies should be mean-reverting. A bad year typically results from cheap stocks getting cheaper.[21] However, a bad year also probably leads the investor to question whether the manager actually *has* skill. In practice, most investors behave as though learning about a manager's skill overwhelms any belief they may have in mean-reversion. That is, money tends to be allocated toward strategies that have performed well lately, indicating that good performance causes them to raise their estimates of future performance. People behave as though active management has momentum and does not mean-revert. This will tend to make the expectations error less persistent and therefore shorten the appropriate horizon for active management. On the

[21] To some extent, I am reflecting my experience with value strategies. It is less so for many other strategies. However, I believe it is the case that most managers would assert that good performance actually lowers their future expectations (and the converse).

other hand, if active manager's returns are mean-reverting, this process systematically moves money away from managers likely to do well.

It might seem as though the horizon need be only as long as it takes to learn whether managers are skillful. Any bad ones will presumably not last. But this is not true for three reasons. First of all, because there is a significant amount of luck involved, it is surprisingly hard to distinguish good managers from bad ones. This is compounded by the fact that "skill" – the ability to add value – varies with the investment environment and may change over time for other reasons as well. Second, a little learning can be a dangerous thing. If active alpha is actually mean-reverting but investors fire underperforming (replacing them with those who have recently outperformed), the chances are they'll see their alpha erode. Finally, even if underperforming managers are not retained, the process of selecting new managers will presumably be similar to the one that led to the selection of the old ones. (A given investor will tend to prize and shun the same manager traits consistently.[22])

At the same time, good managers' skill may not persist. Both managers and the market change over time. It is difficult to maintain a high level of skill over long periods of time. I think the best approach to time horizon is to use a long one, reflecting the time to the ultimate use of the funds, but to allow for less than perfect correlation in skill from year to year. That is, perhaps assume that the correlation between skill in one year and the next is 0.9 instead of 1.0. This will mute somewhat the impact of expectations risk in the long run.

Assumptions also need to be made about the correlation matrix of expectations risk between the various investment alternatives. The variability risk correlation matrix is probably a good place to start. However, there is reason to think that ambiguity correlations are higher than that, especially between active strategies. As I noted before, the managers are being selected by the same person or group. In addition, questions such as how efficient the market is will affect all active managers similarly. Therefore, it seems reasonable to use correlations for ambiguity that are higher than those for variability risk.

10. Conclusion

We cannot know expected returns of the investments we are considering; we can only estimate them. This uncertainty typically seems small relative to the obvious risk around the expected return and, for short horizons, it is. However, as we have seen, errors in expected return estimates can lead to large risks over longer time horizons. For example, it makes relatively little difference whether the mean return on the market is 6, 8 or 10% since the actual return can easily be anywhere from -20 to +40. However, in the long run, the random fluctuations will balance out and the difference between 6% growth and 10% growth is enormous. Ignoring this risk can lead us to allocate excessive amounts to investments that have large amounts of ambiguity. I have argued that active management, in general, and hedge funds, in particular, are most subject to this risk.[23]

This paper has outlined a framework for incorporating ambiguity into the allocation process. Users of such a framework need to assess their confidence in the various forecasts they are making. While this demands additional work, it is effort well spent. In

[22] Of course, individuals can learn and institutions do change. But relying on this seems imprudent.
23 Alternatively, it often leads people not to use systematic frameworks for these decisions (or set arbitrary constraints, which amounts to the same thing).

fact, the process of going through all of the key assumptions being made and assessing the confidence one has in each of those assumptions is beneficial in its own right.

Expectations risk surely exists.[24] Recognizing it should lead to a better assessment of long-term risk and better investment decisions. In particular, it may allow for allocations to active strategies that are both more systematically arrived at and more reasonable than are typical today. [25]

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24 This was driven home when I discovered that even coin tosses may not be fair. See Diaconis (2004). Although, John Von Neuman devised a way to assure equal odds even with a biased coin.

[25] Bernstein Investment Research and Management, A Unit of AllianceBernstein, L.P. has implemented a process based on many of these ideas.

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